

# Optimal trading policies for wind energy producer

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## Abstract

We study the optimal trading policies for a wind energy producer who aims to sell the future production in the open forward, spot, intraday and adjustment markets, and who has access to imperfect dynamically updated forecasts of the future production. We construct a stochastic model for the forecast evolution and determine the optimal trading policies which are updated dynamically as new forecast information becomes available. Our results allow to quantify the expected future gain of the wind producer and to determine the economic value of the forecasts.

Key words: wind energy, forecasts, optimal trading policies, stochastic control

## 1 Introduction

Wind power is now widely recognized as an important part of the global energy mix, and the actors of the energy industry have no choice but to cope with the intermittent and to a large extent unpredictable nature of the wind power production. In particular, as the guaranteed purchase schemes are either phased out or replaced with more market-oriented subsidies, the wind power producers face the need to sell the future power production in the open markets in the absence of precise knowledge of the volume to be produced. The need of wind power producers to adjust their delivery volume estimates as the forecast becomes more precise is one of the factors behind the development of intraday electricity markets, at which power can be traded up to 45 minutes prior to delivery.

The aim of this paper is to determine the optimal strategies for selling the future power production of a single wind park for a wind producer who has access to imperfect dynamically updated forecast of the future production, which becomes progressively more precise as the production horizon draws near. We formulate this problem as a stochastic optimization problem where the power producer aims to maximize the expected gain from selling electricity penalized by terms accounting for market illiquidity and the extra cost of using the adjustment market. To solve this problem, we develop a stochastic model for the forecast evolution, and determine the optimal trading strategy which is updated dynamically as new forecast information becomes available. This

allows to quantify the optimal expected gain for the producer, and to compare the expected gain under different assumptions on the forecast dynamics, thus quantifying the economic value of different forecasts.

Wind power producers in Europe and in many other countries with deregulated energy sector have access to four types of markets.

- The forward market – more than 1 day prior to delivery, delivery periods are day, week, month, quarter and year.
- Spot market – 1 day prior to delivery, delivery period is 1 hour or 30 minutes.
- Intraday market – between 1 day and 45 min, delivery period is 15 minutes.
- Adjustment (imbalance) market (usually managed by the power network operator such as RTE in France) – the last 45 minutes. In the adjustment market, the bid-ask spread is very wide, which may be interpreted as a penalty imposed on the agents for using this market.

Optimal trading strategies for wind power producer with a focus on intraday markets have been considered by several authors. Morales et al. [8] consider the short-term trading for a wind power producer and determine the optimal strategies starting from a small number of scenarios of wind power production generated with an autoregressive model, without taking into account the available forecasts. Henriot [7] studies optimal design of intraday markets in the presence of wind power producers who use certain pre-determined strategies (without optimization). Garnier and Madlener [6] show how forecast errors may be corrected by optimal trading in intraday markets. The paper which is closest in spirit to ours is Aïd et al. [1]. These authors consider the optimal trading problem in intraday markets in the presence of imperfect demand forecasts and market impact, however, unlike our paper they do not focus on wind energy.

The rest of the paper is structured as follows. In section 2 we study the realized production data and show that the distribution of the realized production is well described with a truncated log-normal distribution. Section 3 focuses on forecast dynamics: using some ideas from financial mathematics, we develop a stochastic model for the forecast evolution which is compatible with the truncated log-normal distribution for the realized production. Finally, in Section 4, we formulate and solve in several different settings, relevant for large and small power producers, the optimization problem for the wind power producer who aims to maximize the expected gain from selling the future production.

## 2 Modeling the realized production

We define the normalized output power of a wind park  $F_T$  by

$$F_T = 0 \vee \frac{P_T}{P_{\max}},$$

where  $P_T$  is the actual instantaneous power production (in practice the instantaneous production will be replaced with 10-minute average), and  $P_{\max}$  is the rated power of

the park. Since some of the turbine equipment consumes power, the actual realized power production may sometimes have small negative values; to remove this effect, the normalized power output is truncated from below by 0.

To build a model for the normalized output power, we assume that  $F_T$  is obtained by applying a “stylized power curve”  $f_{prod}$  to the “stylized wind speed”  $X_T$ :

$$F_T = f_{prod}(X_T)$$

We emphasize that the model is built for the output power directly and not for the wind; the power curve and wind speed are introduced merely to provide a rationale for the model. The stylized wind speed  $X_T$  follows a log-normal distribution with parameters  $\mu_X$  and  $\nu_X$ , whose density is

$$\rho_X(x) = \frac{1}{x\sqrt{2\pi\nu_X}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu_X}{\nu_X}\right)^2\right).$$

We assume that the variable  $X_T$  follows a log-normal distribution because:

- The log-normal distribution has been used in the literature as a model for wind speeds [5]. It is also quite close to the Weibull distribution, which is the parametric model of choice for wind speed data;
- The log-normal distribution is analytically tractable and allows to introduce a dynamical aspect into the model via a Brownian motion.

The stylized production function is

$$f_{prod}(x) = \frac{(x - x_{min})^+ - (x - x_{max})^+}{x_{max} - x_{min}}.$$

This shape of this function is illustrated in Figure 1; note that there is no cut-out.

The above assumptions imply that  $F_T$  follows a truncated log-normal distribution with parameters

$$\begin{cases} \zeta = -\frac{x_{min}}{x_{max} - x_{min}} \\ \mu = \mu_X - \ln(x_{max} - x_{min}) \\ \nu = \nu_X. \end{cases} \quad (1)$$

On the interval  $(0, 1)$  this distribution is absolutely continuous with density given by

$$\begin{cases} \rho_F(y|\mu, \nu, \zeta) = \frac{1}{(y - \zeta)\sqrt{2\pi\nu}} \exp\left(-\frac{(\ln(y - \zeta) - \mu)^2}{2\nu^2}\right). \end{cases} \quad (2)$$

In addition, at points 0 and at 1 the distribution has atoms given by

$$\mathbb{P}[F_T = 0] = \mathbb{P}(X_T \leq x_{min}) = \Phi\left(\frac{\ln x_{min} - \mu_X}{\nu_X}\right) = \Phi\left(\frac{\ln(-\zeta) - \mu}{\nu}\right) := P_0(\mu, \nu, \zeta)$$

$$\mathbb{P}[F_T = 1] = \mathbb{P}(X_T > x_{max}) = 1 - \Phi\left(\frac{\ln(1 - \zeta) - \mu}{\nu}\right) := P_1(\mu, \nu, \zeta).$$

Note that while the original construction used four parameters  $(\mu_X, \nu_X, x_{min}, x_{max})$ , one parameter is redundant, and the distribution of  $F_T$  is completely characterized by the three parameters  $\mu, \nu, \zeta$ . To remove this redundancy, we shall set  $\mu_X = -\frac{1}{2}\nu_X^2$  in the following, which ensures that  $\mathbb{E}[X_T] = 1$ .

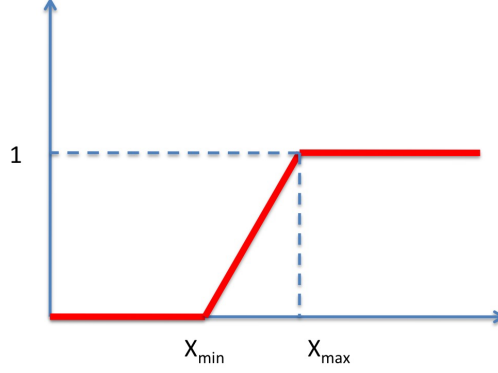


Figure 1: Stylized power curve used to model the realized production

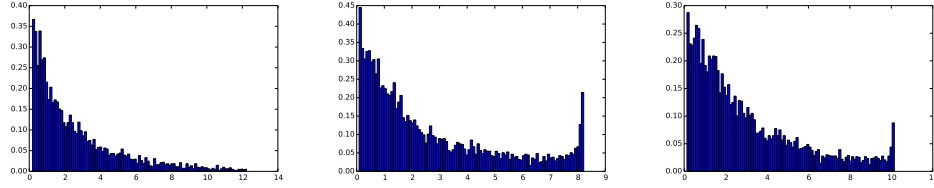


Figure 2: Histograms of 10-minute realized power production, with 4-hour subsampling, for the three power plants which are the object of this study, excluding the atom at zero.

**Fitting the model** The model was fitted to the output power at the wind park level for 3 wind parks in France, sampled at 10-minute intervals from Jan 1st, 2011 to Jan 1st, 2015, provided by the company Maïa Eolis (hereafter referred to as Plant 1, Plant 2 and Plant 3). Figure 2 shows the histograms of the realized production for the three plants (plants are numbered from left to right in this and other graphs).

Denote the observed normalized output power values by  $(F_T^k)_{k=1}^N$ , and assume that they are arranged in *increasing order*. The method consists in minimizing the Euclidean distance between the empirical quantiles and the quantiles of the theoretical distribution. More precisely, given  $\alpha \in [0, 1]$ , we define the empirical quantile

$$q_{emp}^\alpha = \max \left\{ F_T^k \left| \frac{k}{N} \leq \alpha \right. \right\}, \quad (3)$$

and, for  $P_0(\mu, \nu, \zeta) \leq \alpha \leq 1 - P_1(\mu, \nu, \zeta)$ , we define the theoretical quantile

$$q^\alpha(\mu, \nu, \zeta) = \max \left\{ x \left| \Phi \left( \frac{\ln(x - \zeta) - \mu}{\sigma} \right) \leq \alpha \right. \right\},$$

where  $\Phi$  is the standard normal distribution function. The parameters are estimated by

| Parameters     | Plant 1  | Plant 2  | Plant 3  |
|----------------|----------|----------|----------|
| $\mu$          | -1.46551 | -0.60213 | -0.76199 |
| $\sigma$       | 0.66020  | 0.46158  | 0.48778  |
| $\zeta$        | -0.13248 | -0.33757 | -0.26449 |
| $x_{min}$      | 0.46129  | 0.55412  | 0.50312  |
| $x_{max}$      | 3.94322  | 2.19561  | 2.40534  |
| $\mu_{X_T}$    | -0.21793 | -0.10653 | -0.11896 |
| $\sigma_{X_T}$ | 0.66020  | 0.46158  | 0.48778  |

Table 1: Fitted parameters of normalized production and the corresponding parameters  $(x_{min}, x_{max}, \mu_{X_T}, \sigma_{X_T})$ .

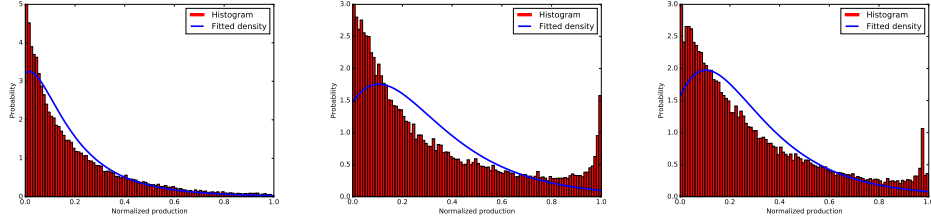


Figure 3: Fitted vs. empirical densities for the three power plants. The total production has been normalized to one.

minimizing

$$\sum_{l=1}^L \left( q_{emp}^{\alpha_l(\mu, \nu, \zeta)} - q^{\alpha_l(\mu, \nu, \zeta)}(\mu, \nu, \zeta) \right)^2,$$

where  $(\alpha_l)_{l=1}^L$  are probability levels, uniformly spaced between  $P_0(\mu, \nu, \zeta)$  and  $1 - P_1(\mu, \nu, \zeta)$ , that is,

$$\alpha_l = P_0(\mu, \nu, \zeta) + \frac{l-1}{L}(1 - P_1(\mu, \nu, \zeta) - P_0(\mu, \nu, \zeta)).$$

In the numerical example below,  $L = 100$  probability levels were used.

Table 1 gives the fitted optimal parameters  $(\mu^*, \nu^*, \zeta^*)$  and the corresponding latent parameters  $(\mu_{X_T}, \nu_{X_T}, x_{min}, x_{max})$  obtained for the three power plants. The fitted truncated log-normal densities are shown in Figure 3.

### 3 Modeling the forecast dynamics

To understand how to optimally update the trading strategy depending on the forecast, we need to build a dynamic stochastic model for the forecast, which is consistent with the distribution of the realized production described in the previous section. More precisely, at every date  $t$  we assume that the forecast  $F_t$  is the best prediction of the realized production given the available information.

To build a forecast model formalizing this idea, we need to define a filtration  $\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq T}$ , where  $\mathcal{F}_t$  models the information available to the wind producer at time  $t$ , and a stochastic process  $(F_t)_{0 \leq t \leq T}$  with the following properties:

- It is a martingale with respect to the filtration  $\mathbb{F}$ ;
- $F_T$  has the truncated log-normal distribution described in the preceding section.

We now proceed with the construction of the filtration and the process  $F$ . Let  $W$  be a standard Brownian motion, and  $Z$  be a standard normal random variable independent from  $W$ . We define the process  $(X_t)_{0 \leq t \leq T}$  by

$$X_t = \exp \left( \int_0^t \sigma(s) dW_s - \frac{1}{2} \int_0^t \sigma^2(s) ds \right), t < T$$

and

$$X_T = \exp \left( \int_0^T \sigma(s) dW_s - \frac{1}{2} \int_0^T \sigma^2(s) ds \right) e^{bZ - \frac{b^2}{2}}.$$

where  $(\sigma(s))_{0 \leq s \leq T}$  is a square integrable deterministic function and  $b \geq 0$ . We then define  $\mathbb{F}$  to be the natural filtration of  $X$  completed with the null sets.

In other words, for each fixed  $t$ ,

$$X_T \stackrel{d}{=} X_t e^{\sqrt{\theta(t)}N - \frac{\theta(t)}{2}} \quad \text{where } N \sim N(0, 1) \quad \text{and} \quad \theta(t) = \int_t^T \sigma(s)^2 ds + b^2,$$

and  $N$  is independent from  $X_t$ . Letting  $(\mathcal{F}_t)_{0 \leq t < T}$  be the completed natural filtration of the Brownian motion  $W$  and  $\mathcal{F}_T := \mathcal{F}_0 \vee \sigma(Z) \vee \sigma(W_s, 0 \leq s \leq T)$ , we see that  $X$  is an  $\mathbb{F}$ -martingale which means that the variable  $X_t$  may be seen as the best prediction of the stylized wind  $X_T$  given the information available at time  $t$ . The jump at time  $T$  is needed to model the component of the wind which is not predictable even at very short time horizons. It is clear that by taking

$$\nu_X = \theta(0),$$

we recover the distribution of  $X_T$  described in the preceding section.

We then define the *forecast process* by

$$F_t = \mathbb{E}[f_{prod}(X_T) | \mathcal{F}_t].$$

The following proposition gives an explicit form of this process.

**Proposition 1.** *The forecast process is given explicitly by*

$$F_t = g(X_t, \theta(t)),$$

where

$$g(x, \theta) = \frac{1}{x_{max} - x_{min}} [x(\Phi(d_+^{min}(x, \theta)) - \Phi(d_+^{max}(x, \theta))) - x_{min}\Phi(d_-^{min}(x, \theta)) + x_{max}\Phi(d_-^{max}(x, \theta))]$$

with  $d_{\pm}^{min, max}(x, \theta) = \frac{1}{\sqrt{\theta}} [\ln(x/x_{min, max}) \pm \theta/2]$  and  $\Phi$  is the standard normal distribution function.

This model fully describes the evolution of the forecast dynamics, while ensuring that  $F_t(T) \in [0, 1]$  for all  $t$ . For every forecast horizon, the forecast distribution is parameterized by a single number,  $\theta(t)$ . Since the key quantity for determining the optimal strategy is the forecast error, we fit the function  $\theta$  by matching the empirically observed variances of the forecasting errors for different horizons with the variances predicted by the model and given by

$$\begin{aligned}\mathbb{E}[(F_t - F_T)^2] &= \mathbb{E}[(g(\theta(t), X_t) - f_{prod}(X_T))^2] \\ &= \mathbb{E}[f_{prod}(X_T)^2] - \mathbb{E}[g(\theta(t), X_t)^2].\end{aligned}$$

Computing the second term requires a one-dimensional numerical integration:

$$\mathbb{E}[g(\theta(t), X_t)^2] = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} g\left(\theta(t), \exp\left(\sqrt{\nu_X^2 - \theta(t)}z - \frac{\nu_X^2 - \theta(t)}{2}\right)\right)^2 e^{-\frac{z^2}{2}} dz.$$

As for the first term, it may be evaluated explicitly:

$$\begin{aligned}\mathbb{E}[f_{prod}(X_T)^2] &= \Phi(d_0^{max}) + \frac{e^{\frac{\nu_X^2}{2}}}{(x_{max} - x_{min})^2} \{\Phi(d_0^{min}) - \Phi(d_0^{max})\} \\ &\quad - \frac{2x_{min}}{(x_{max} - x_{min})^2} \{\Phi(d_+^{min}) - \Phi(d_+^{max})\} \\ &\quad + \frac{x_{min}^2}{(x_{max} - x_{min})^2} \{\Phi(d_-^{min}) - \Phi(d_-^{max})\},\end{aligned}$$

where

$$d_0^{max,min} = \frac{-\log x_{max,min} + \frac{3\nu_X^2}{2}}{\nu_X}, \quad d_{\pm}^{max,min} = \frac{-\log x_{max,min} \pm \frac{\nu_X^2}{2}}{\nu_X}.$$

Therefore, for fixed parameters of the realized production distribution  $\nu_X, x_{min}$  and  $x_{max}$ , the forecast error variance  $\mathbb{E}[(F_t - F_T)^2]$  is a function of  $\theta(t)$  only. For  $\theta \in [0, \nu_X^2]$ , let

$$\phi(\theta) = \mathbb{E}[f_{prod}(X_T)^2] - \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} g\left(\theta, \exp\left(\sqrt{\nu_X^2 - \theta}z - \frac{\nu_X^2 - \theta}{2}\right)\right)^2 e^{-\frac{z^2}{2}} dz.$$

By Jensen's inequality it can be shown that  $\phi(\theta)$  is strictly increasing in  $\theta$ . Moreover it is clearly continuous and satisfies  $\phi(0) = 0$  and  $\phi(\nu_X^2) = \text{Var}[f_{prod}(X_T)]$ . Therefore, for any  $v$  in the interval  $(0, \text{Var}[f_{prod}(X_T)])$ , there exists a unique  $\theta$  such that  $\phi(\theta) = v$ . We use this property to calibrate the function  $\theta(\cdot)$  non-parametrically to the observed variances of the forecast errors.

Alternatively, one can use a parametric volatility function given by

$$\sigma_t = \sigma_0 e^{\eta(T-t)} \mathbf{1}_{t > T-\tau^*}.$$

Here,  $\tau^*$  is the time horizon for which the forecast error variance becomes equal to the unconditional variance of the realized power output, which means that the forecast becomes useless. This corresponds to the function  $\theta(t)$  given by

$$\theta(t) = \left\{ b^2 + \frac{\sigma_0^2}{2\eta} \left( e^{2\eta(T-t)} - 1 \right) \right\} \wedge \nu_X^2.$$

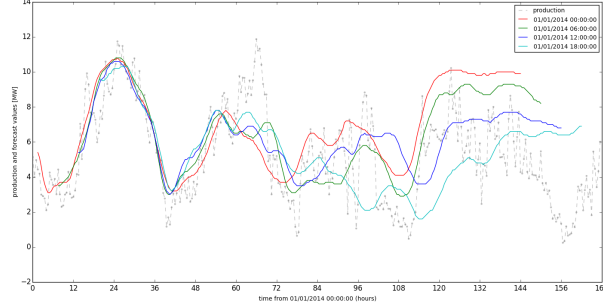


Figure 4: Plot of the forecast made at a given date as function of time horizon together with the realized production for this horizon for four different starting times (given in the legend). Accuracy decreases for longer horizons.

**Fitting the model** We estimate the function  $\theta(\cdot)$  in both the non-parametric and the parametric form using the forecast data provided by Maïa Eolis. This data set contains the forecasts of the power output at wind park level, produced by an independent forecasting company, for the period from December 7th 2011 to March 3rd 2015. In the numerical examples we focus on the wind park 1 from the three parks considered in the previous section. The forecasts are updated every 6 hours and cover time horizons from 1h15min to 144 hours ahead with 15 minute step. The forecast values are positive, and in the analysis we normalize them by the rated power of the plant so that  $F_t(T) \in [0, 1]$ . Figure 4 shows examples of forecasts together with the actual realized production. The forecasts appear quite precise for short time horizons, but the precision deteriorates significantly for longer horizons. This is further confirmed in Figure 5 which shows the histograms of the forecast errors for different horizons.

Figure 6, left graph, plots the variance of the forecast error as function of time horizon. More than  $\tau^* = 120h$  prior to production date the variance of the forecast error exceeds that of the realized production and we consider that the forecast has no value. The right graphs of this figure shows the function  $\theta$  estimated using the non-parametric method described above and the parametric method (when all error variances are fitted at the same time by nonlinear least squares). The estimated parameter values are  $\sigma_0 = 0.040113$ ,  $\eta = 0.004423$  and  $b = 0.308817$ .

Finally, Figure 7 compares the empirical distribution of the forecast error with the one generated by the model for the time horizon of 48 hours.

## 4 Optimization of market interventions

In this section, our aim is to determine the optimal strategies for selling electricity produced during a short time period  $[T - \delta, T]$ , where  $T$  is fixed. This electricity must be sold in advance, in different markets (spot, forward, intraday), otherwise a penalty



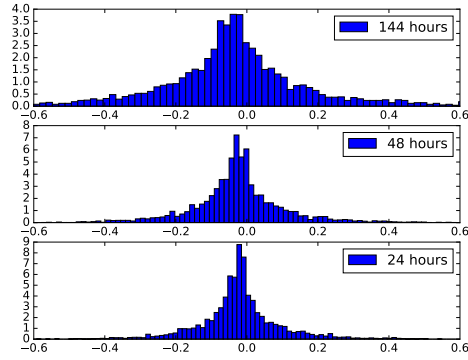


Figure 5: Histograms of the forecast error for different time horizons.

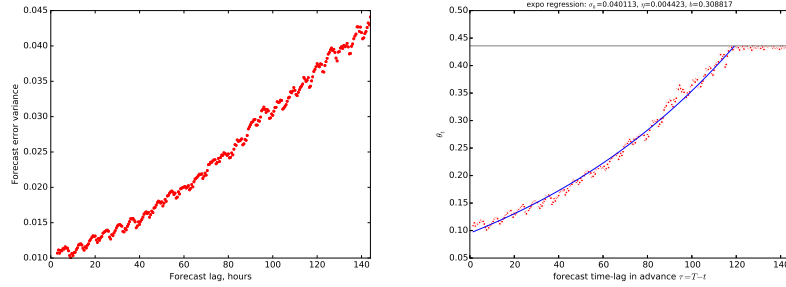


Figure 6: Left: variance of the forecast error as function of time horizon. Right: function  $\theta$  estimated using parametric and nonparametric method. The 6-hour periodicity is due to the fact that forecast is updated every 6 hours.

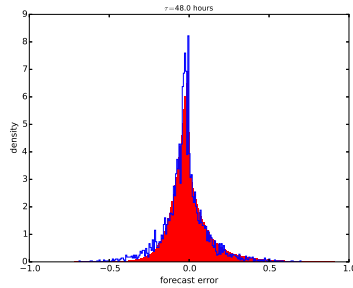


Figure 7: The histogram of the observed forecasting error (blue line) compared to the model-generated histogram for the time horizon of 48 hours (red bars).

is applied for using the adjustment market. We assume that the wind power producer does not know the exact production but has forecasts available.

There are several reasons for trading both in intraday and spot/forward markets. First of all, intraday markets are very volatile and illiquid: if supply exceeds demand, the prices plunge down and if demand exceeds supply, the prices shoot up (see Figure 9). This means that large amounts of energy can only be sold at a very low price. For this reason, it is advantageous to sell in spot / forward markets if the amount of electricity to be produced is known in advance. Also, by selling in the forward market, one reduces the risk associated to the change in the price until the delivery date, since forward prices fluctuate less than spot / intraday prices. On the other hand, selling in the spot/intraday market reduces the penalty applied for not delivering the right amount since the forecasts are better when the delivery horizon is close.

**Forward price model** In practice, the forward contracts are traded continuously but cover an extended delivery period, e.g., one year, one quarter, one month, one week and sometimes one day. In the spot market, trading takes place only once per day, and one can make separate bids for each hour of the following day. The intraday market again allows continuous trading and the basic contract covers a 15-minute delivery period. To simplify the treatment and make our main ideas transparent, we do not distinguish between different markets, and assume that at every time  $t$ , one can enter into a forward contract allowing to buy / sell electricity at a future date  $T$  at the price  $P_t(T)$ . Our methods and results can be easily adapted to a more realistic market structure.

Let  $\mathbb{G} := (\mathcal{G}_t)_{t \geq 0}$  be the filtration of the agent selling electricity. We assume that the forward sale price process satisfies

$$dP_t(T) = \mu_t dt + \beta_t dB_t,$$

where  $\mu$  and  $\beta$  are deterministic processes such that

$$\int_0^T (|\mu_t| + \beta_t^2) dt < \infty$$

and  $B$  is a  $\mathbb{G}$ -Brownian motion. For longer horizons, the coefficient  $\mu$  reflects the average trend of forward prices as the delivery horizon draws near. As seen from Figure 8, this trend is typically negative, which corresponds to a premium for early trading. For shorter time horizons the negative coefficient  $\mu$  may reflect the widening of the bid-ask spread in the intraday market.

**Volume penalty** We assume that the wind power producer has the obligation to sell all the produced energy and denote by  $\phi_t$  the aggregate position at time  $t$  (total quantity to deliver at time  $T$  owing to the contracts entered into prior to date  $t$ ). The trading starts at some fixed date 0. If, at date  $T$ ,  $\phi_T \neq F_T$ , the agent must sell / purchase the extra energy at price  $P_T := P_T(T)$ , and in addition pay a penalty equal to  $u(F_T - \phi_T)$ , where  $u(0) = 0$ ,  $u(x)$  is increasing for  $x > 0$  and decreasing for  $x < 0$ .

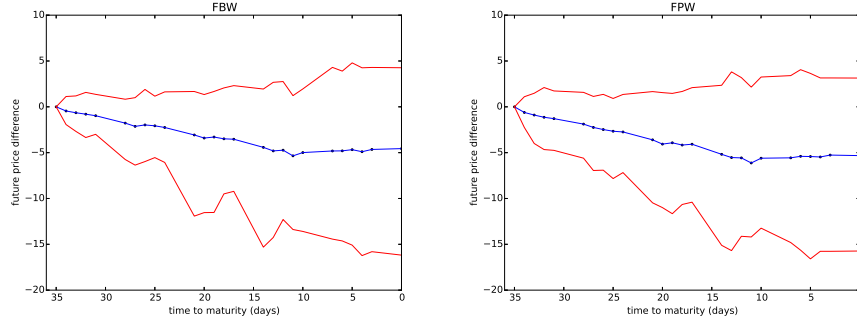


Figure 8: Evolution of future price difference  $P_t(T) - P_0(T)$  as function of  $t$ , averaged over one year, with 95% confidence bounds. Left: base futures. Right: peak futures.

**Admissible strategies** We are interested in determining the optimal strategies for two kinds of electricity producers: a small producer whose interventions do not affect market prices, and a relatively large producers whose trades may impact the market. The small producer is only selling the electricity and does not engage in proprietary trading. Therefore, the class  $\mathcal{A}$  of admissible strategies for a small producer contains all  $\mathbb{G}$ -adapted increasing processes  $\phi$  with  $\phi_0 = 0$  satisfying the condition

$$\mathbb{E} \left[ \int_0^T (\phi_t |\mu_t| + \phi_t^2 \beta_t^2) dt \right] < \infty.$$

Indeed, allowing  $\phi$  to both increase and decrease does not make sense in the absence of market impact since in that case the optimal strategy would be to sell all produced electricity just before the terminal date.

For the large producer, following [2], we assume that the trading strategy is absolutely continuous and introduce a market impact term proportional to the square of the rate of trading  $\psi_t = \phi'_t$ . The class  $\mathcal{A}_+^{ac}$  of admissible strategies for a large producer who can only sell electricity thus contains all processes in  $\mathcal{A}$  which are absolutely continuous. Finally, the class  $\bar{\mathcal{A}}^{ac}$  of admissible strategies for a large producer who can both buy and sell electricity contains all processes of the form  $\phi = \phi^+ - \phi^-$  where  $\phi^+$  and  $\phi^-$  are in  $\mathcal{A}_+^{ac}$ .

**Gain from trading** For a small producer whose transactions do not create market impact, the total gain from selling electricity is modeled (in continuous time) by

$$G = F_T P_T - \int_0^T \phi_t dP_t(T) - u(F_T - \phi_T),$$

where  $\phi \in \mathcal{A}$ . For a large producer, with the penalization by the market impact term, the gain from trading becomes

$$G = F_T P_T - \int_0^T \phi_t dP_t(T) - u(F_T - \phi_T) - \frac{\gamma}{2} \int_0^T \psi_s^2 ds.$$

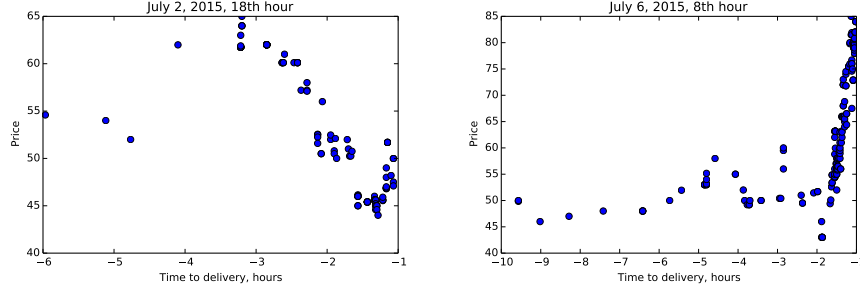


Figure 9: Intraday transaction prices for a fixed delivery hour. As the delivery horizon draws close, volatility increases and bid-ask spreads widen.

where  $\phi \in \mathcal{A}^{ac}$ . In the following sections we consider separately the problems of maximizing the expected gain with and without market impact.

#### 4.1 Trading for a small producer in absence of market impact

The optimization problem for trading in the absence of market impact writes

$$\min_{\phi \in \mathcal{A}} \mathbb{E} \left[ \underbrace{\int_0^T \phi_t \mu_t dt}_{\text{Expected loss from trading early (negative)}} + \underbrace{u(F_T - \phi_T)}_{\text{volume penalty}} \right]. \quad (4)$$

Note that the stochastic part of the price process does not play a role in this optimization problem, and the solution is therefore independent from the price volatility. One could introduce a risk penalty to account for the price volatility effects but we do not pursue this here.

Before obtaining a general solution with numerical methods, we first present explicit solutions in the cases where the forecast information is either exact or unavailable. This will allow us to establish upper and lower bounds on the expected gain which may be obtained with probabilistic forecast.

**Exact forecast** Assume that the future realized production is known without error, in other words,  $F_T \in \mathcal{G}_0$ . In this case, the optimal strategy is described by the following proposition, where we define

$$t^* = \arg \min_{0 \leq t \leq T} \int_t^T \mu_s ds, \quad m^* = m_{t^*} = \int_{t^*}^T \mu_s ds.$$

**Proposition 2.** *Let the penalty function  $u$  be convex and continuously differentiable, with  $u'(0) = 0$  and  $\lim_{x \rightarrow -\infty} u'(x) = -\infty$ . Then the value function of the the problem*

(4) is given by

$$\begin{aligned} F_T m^* - v(m^*), & \quad m^* < 0; \\ -v(0) \equiv u(0), & \quad \text{otherwise,} \end{aligned}$$

where  $v(y)$  is the Fenchel transform of  $u$ :  $v(y) = \sup_x \{xy - u(x)\}$ .

Denote by  $I(y)$  the inverse function of  $u'$ . The optimal strategy for the problem (4) is described as follows.

- If  $m^* \leq 0$ , sell the quantity  $\phi = F_T - I(m^*)$  at time  $t^*$ .
- If  $m^* > 0$ , sell at time  $T$  the amount  $F_T$ .

*Remark 1.* Although the realized production is known in advance, sometimes it is advantageous for the agent to sell more than the realized production, to be able to benefit from the higher prices in the beginning of trading.

*Proof.* We first transform the optimization functional with an integration by parts.

$$\int_0^T \phi_t \mu_t dt + u(F_T - \phi_T) = \int_0^T \left( \int_t^T \mu_s ds \right) d\phi_t + u(F_T - \phi_T).$$

It is now clear that for fixed  $\phi_T$ , the optimal solution  $(\phi_t)_{0 \leq t \leq T}$  is such that the measure  $d\phi$  is supported by the single point  $t^*$ . Therefore,  $\phi_t = \phi \mathbf{1}_{t \geq t^*}$  with a constant  $\phi$ . If  $m^* > 0$ , it is optimal to choose  $t^* = T$  and  $\phi$  which minimizes  $u(F_T - \phi)$ , that is  $\phi = F_T$ . Otherwise,  $\phi$  can be found by solving the optimization problem

$$\min_{\phi \geq 0} [\phi m^* + u(F_T - \phi)].$$

The candidate optimizer is given by  $\phi = F_T - I(m^*)$ . It is easy to check from our assumption that this quantity is positive, which means that

$$\min_{\phi \geq 0} [\phi m^* + u(F_T - \phi)] = \min_{\phi \in \mathbb{R}} [\phi m^* + u(F_T - \phi)] = v(m^*) + F_T m^*.$$

□

**Absence of forecast** In this case, we assume that the agent does not have access to the forecast but only knows the distribution of power production, in other words,  $(\mathcal{G}_t)_{0 \leq t < T}$  coincides with the completed natural filtration of  $B$  and  $\mathcal{G}_T$  in addition contains  $F_T$ . The agent's strategy is then deterministic on  $[0, T)$  with a possible random jump at time  $T$  (when the realized production becomes known). The optimal strategy is described by the following proposition, where we define  $\bar{u}(x) = \mathbb{E}[\bar{u}(F_T - \mathbb{E}[F_T] + x)]$  and  $\bar{u}(x) = u(x)$  if  $x \leq 0$  and  $\bar{u}(x) = u(0)$  if  $x > 0$ .

**Proposition 3.** Let  $\mathbb{E}[|F_T|] < \infty$  and assume that the penalty function  $u$  satisfies the assumptions of Proposition 2 and in addition

$$\mathbb{E}[u(F_T + x)] < \infty \quad \text{and} \quad \mathbb{E}[|u'(F_T + x)|] < \infty \quad \forall x \in \mathbb{R}.$$

Then the value function of the the problem (4) is given by

$$\begin{aligned} & \mathbb{E}[F_T]m^* - \tilde{v}(m^*), & m^* < 0; \\ & u(0), & \text{otherwise,} \end{aligned}$$

where  $\tilde{v}(y) = \sup_x \{xy - \tilde{u}(x)\}$ . The optimal strategy is described as follows (we denote the inverse function of  $\tilde{u}'$  by  $\tilde{I}$ ).

- If  $m^* \leq 0$ , sell the quantity  $\phi = \mathbb{E}[F_T] - \tilde{I}(m^*)$  at time  $t^*$  then sell  $F_T - \mathbb{E}[F_T] + \tilde{I}(m^*)$  (if this quantity is positive) at time  $T$ .
- If  $m^* > 0$ , sell the quantity  $F_T$  at time  $T$ .

*Proof.* Using the assumptions on  $u$ , by the dominated convergence theorem, we can show that  $\tilde{u}$  is convex and continuously differentiable. Similarly to the proof of Proposition 2, we find that when  $m^* < 0$ , the optimal strategy has the form

$$\phi \mathbf{1}_{t \geq t^*} + (F_T - \phi) \mathbf{1}_{t \geq T},$$

where  $\phi$  is found by solving the optimization problem

$$\min_{\phi \geq 0} \{\phi m^* + \tilde{u}(\mathbb{E}[F_T] - \phi)\}.$$

The candidate optimizer is given by  $\phi = \mathbb{E}[F_T] - \tilde{I}(m^*)$ . From our assumptions it follows that  $\phi$  is nonnegative, and therefore

$$\min_{\phi \geq 0} \{\phi m^* + \tilde{u}(\mathbb{E}[F_T] - \phi)\} = \min_{\phi \in \mathbb{R}} \{\phi m^* + \tilde{u}(\mathbb{E}[F_T] - \phi)\} = \mathbb{E}[F_T]m^* - \tilde{v}(m^*).$$

□

*Example 1.* Assume that the penalty is quadratic, that is,  $u(x) = \frac{\kappa}{2}x^2$  and  $\bar{u}(x) = \frac{\kappa}{2}x^2 \mathbf{1}_{x < 0}$ , and the realized production  $F_T$  is uniformly distributed on  $[0, 1]$ . Then

$$\tilde{u}(x) = \frac{\kappa}{6} \left( \frac{1}{2} - x \right)^3 \mathbf{1}_{-\frac{1}{2} \leq x \leq \frac{1}{2}} + \frac{\kappa}{2} \left( x^2 + \frac{1}{12} \right) \mathbf{1}_{x < -\frac{1}{2}}$$

and thus

$$\tilde{I}(z) = \begin{cases} \frac{z}{\kappa} & z < -\frac{\kappa}{2} \\ \frac{1}{2} - \sqrt{-\frac{2z}{\kappa}} & z \geq -\frac{\kappa}{2} \end{cases}$$

**Discrete forecast updates** In this paragraph we consider the more realistic situation when the forecast is updated at a finite set of deterministic times  $0 = t_0 < t_1 < \dots < t_n = T$ , that is,

$$F_t = \sum_{i=0}^{n-1} F_i \mathbf{1}_{t_i \leq t < t_{i+1}} + F_n \mathbf{1}_{t_n \leq t},$$

where  $(F_k)$  is a discrete-time martingale with respect to the discrete-time filtration  $\mathcal{F}_k = \sigma(F_i, 0 \leq i \leq k)$ . Moreover, we make the assumption that  $\mu_t \leq 0$  for  $t \in [0, T]$ , that is, the expected price may only fall as the delivery date approaches. We denote

$$m_k = \int_{t_k}^{t_{k+1}} \mu_s ds.$$

The optimal strategy is described by the following proposition.

**Proposition 4.** *Let the penalty function  $u$  satisfy the assumptions of Proposition 3 and assume in addition that*

$$\mathbb{E}[|u'(x - c(F_0 + \dots + F_n))|] < \infty$$

*for all  $x \in \mathbb{R}$  and some constant  $c > 1$ . Then there exists a discrete-time  $(\mathcal{F}_k)$ -adapted process  $(\xi_k)_{0 \leq k \leq n}$  such that*

$$\sum_{i=k}^{n-1} m_i = \mathbb{E}[u'(F_n - \max_{k \leq i \leq n} \xi_i) | \mathcal{F}_k] \quad (5)$$

*for  $k = 0, \dots, n$ . The optimal trading strategy is given by*

$$\phi_t = \sum_{i=0}^{n-1} \phi_i \mathbf{1}_{t_i \leq t < t_{i+1}} + \phi_n \mathbf{1}_{t_n \leq t}, \quad (6)$$

*where*

$$\phi_k = \max_{0 \leq i \leq k} \xi_i$$

*for  $0 \leq k \leq n$ .*

*Remark 2.* The process  $(\xi_k)$  may be computed by backward induction. This proposition can be extended to the continuous-time case using the results of [3], following, e.g., [4]. However, the discrete-time case is more relevant in practice, since the forecasts are updated in discrete time. In addition, for numerical computations time must be discretized anyway. For this reason we concentrate on the discrete case in this paper.

*Proof.* We first prove the existence of the process  $(\xi_k)$  by an induction argument. Clearly, one may choose  $\xi_n = F_n$ . Assume now that for some  $m \leq n$ , we have constructed a process  $(\xi_k)_{m \leq k \leq n}$  satisfying (5) for  $m \leq k \leq n$ , and such that in addition

$$0 \leq \xi_k \leq cF_k - I \left( \frac{c}{c-1} \sum_{i=k}^{n-1} m_i \right) \quad (7)$$

*for  $m \leq k \leq n$ .*

Consider a random function

$$\xi \mapsto f_m(\xi) = \mathbb{E}[u'(F_n - \max(\xi, \max_{m \leq i \leq n} \xi_i)) | \mathcal{F}_{m-1}].$$

This function is well defined and a.s. continuous for all  $\xi \in \mathbb{R}$  by assumptions of the proposition and estimate (7). Remark that

$$\lim_{\xi \rightarrow \infty} f_m(\xi) = -\infty$$

and by the induction hypothesis,

$$f_m(0) = \sum_{i=m}^{n-1} m_i.$$

Therefore, there exists  $\xi_m \in \mathcal{F}_m$  with  $\xi_m \geq 0$ , which solves the equation  $f_m(\xi) = \sum_{i=m-1}^{n-1} m_i$ . Moreover, by Markov inequality,

$$\begin{aligned} \mathbb{E}[u'(F_n - \max(\xi, \max_{m \leq i \leq n} \xi_i)) | \mathcal{F}_{m-1}] &\leq \mathbb{E}[u'(F_n - \max(\xi, F_n)) | \mathcal{F}_{m-1}] \\ &\leq \mathbb{E}[u'(\min(F_n - \xi, 0)) \mathbf{1}_{F_n \leq c\mathbb{E}[F_n | \mathcal{F}_{m-1}]} | \mathcal{F}_{m-1}] \\ &\leq \frac{c-1}{c} u'(\min(c\mathbb{E}[F_n | \mathcal{F}_{m-1}] - \xi, 0)) \leq \frac{c-1}{c} u'(c\mathbb{E}[F_n | \mathcal{F}_{m-1}] - \xi). \end{aligned}$$

This shows that

$$\xi_{m-1} \leq c\mathbb{E}[F_n | \mathcal{F}_{m-1}] - I \left( \frac{c}{c-1} \sum_{i=m-1}^{n-1} m_i \right).$$

It remains to show the optimality of the proposed strategy. First, note that since  $\mu_t \leq 0$  for all  $t \in [0, T]$ , with each interval it is optimal to sell electricity as early as possible, so that the optimal strategy has the form (6). Therefore, we need to minimize the discrete-time version of the objective function

$$J(\phi) = \mathbb{E} \left[ \sum_{k=0}^{n-1} \phi_k m_k + u(F_n - \phi_n) \right]$$

over all increasing adapted discrete-time processes  $(\phi_k)_{0 \leq k \leq n}$ . Let  $\phi_k^* = \max_{0 \leq i \leq k} \xi_i$  and let  $(\phi_k)$  be any other admissible strategy. We denote  $\Delta\phi_i = \phi_i - \phi_{i-1}$  for  $i = 1, \dots, n$  and  $\Delta\phi_0 = \phi_0$ , and similarly for  $\Delta\phi_i^*$ . Then,

$$\begin{aligned} J(\phi) - J(\phi^*) &= \mathbb{E} \left[ \sum_{k=0}^{n-1} (\phi_k - \phi_k^*) m_k + u(F_n - \phi_n) - u(F_n - \phi_n^*) \right] \\ &\geq \mathbb{E} \left[ \sum_{k=0}^{n-1} (\phi_k - \phi_k^*) m_k - u'(F_n - \phi_n^*) (\phi_n - \phi_n^*) \right] \\ &= \mathbb{E} \left[ \sum_{i=0}^n (\Delta\phi_i - \Delta\phi_i^*) \mathbb{E} \left[ \sum_{k=i}^{n-1} m_k - u'(F_n - \phi_n^*) | \mathcal{F}_k \right] \right]. \end{aligned}$$

Now, on the one hand, for all  $k$ ,  $\phi_n^* \geq \max_{k \leq i \leq n} \xi_i$ , which means that

$$\mathbb{E} \left[ \sum_{k=i}^{n-1} m_k - u'(F_n - \phi_n^*) | \mathcal{F}_k \right] \leq \mathbb{E} \left[ \sum_{k=i}^{n-1} m_k - u'(F_n - \max_{k \leq i \leq n} \xi_i) | \mathcal{F}_k \right] = 0.$$



On the other hand, if, for some  $k$ ,  $\Delta\phi_k > 0$  then  $\phi_k = \xi_k$  so that

$$\mathbb{E} \left[ \sum_{k=i}^{n-1} m_k - u'(F_n - \phi_n^*) | \mathcal{F}_k \right] = \mathbb{E} \left[ \sum_{k=i}^{n-1} m_k - u'(F_n - \max_{k \leq i \leq n} \xi_i) | \mathcal{F}_k \right] = 0.$$

Since the processes  $\phi$  and  $\phi^*$  are nondecreasing, these observations together with the above estimate imply that

$$J(\phi) - J(\phi^*) \geq 0,$$

which means that  $\phi^*$  is the optimal strategy.  $\square$

## 4.2 Trading for a large producer in presence of market impact

Our aim now is to maximize the expected gain penalized by market impact and volume penalty. The optimization problem therefore takes the following form.

$$\min_{\phi \in \mathcal{A}_+^{ac}} \mathbb{E} \left[ \underbrace{\int_0^T \phi_t \mu_t dt}_{\text{Expected loss from trading early (negative)}} + \underbrace{\frac{\gamma}{2} \int_0^T \psi_s^2 ds}_{\text{Market impact } (\psi = \phi')} + \underbrace{u(F_T - \phi_T)}_{\text{volume penalty}} \right]. \quad (8)$$

When buying electricity is allowed, the set  $\mathcal{A}_+^{ac}$  is replaced with the set  $\mathcal{A}^{ac}$ .

We first consider the situation when the agent is only allowed to sell electricity. As before, we first focus on the degenerate cases when the forecast is either exact or unavailable.

**Proposition 5** (Exact forecast). *Let the penalty function  $u$  be strictly convex and continuously differentiable with  $u'(0) = 0$  and  $\lim_{x \rightarrow -\infty} u'(x) = -\infty$ . Then the optimal strategy for the problem (8) is given by*

$$\bar{\psi}_t = \frac{1}{\gamma} \left( u'(F_T - \bar{\phi}_T) - \int_t^T \mu_s ds \right)^+,$$

where the terminal value  $\bar{\phi}_T$  is the solution of the equation

$$\phi = \frac{1}{\gamma} \int_0^T dt \left( u'(F_T - \phi) - \int_t^T \mu_s ds \right)^+.$$

*Proof.* The Hamiltonian of this optimization problem is

$$H(t, \phi, x, \psi; p) = \left( \mu_t \phi + \frac{\gamma}{2} \psi^2 \right) + p\psi$$

By Pontriagin's principle for deterministic control problems, for each  $t$ , the optimal strategy  $\bar{\psi}_t$  realizes the minimum of  $H(t, \bar{\phi}_t, x, \psi; \bar{p}_t) = \bar{p}_t \psi + \frac{\gamma}{2} \psi^2 + \bar{\phi}_t \mu_t$  over all  $\psi \geq 0$ , where  $\bar{\phi}_t = \int_0^t \bar{\psi}_s ds$  and the function  $\bar{p}_t$  satisfies

$$\frac{d}{dt} \bar{p}_t = -\frac{\partial}{\partial \phi} H(t, \bar{\phi}_t, x, \bar{\psi}_t; \bar{p}_t) = -\mu_t, \quad p_T = -u'(F_T - \bar{\phi}_T)$$

Therefore,

$$\bar{p}_t = -u'(F_T - \bar{\phi}_T) + \int_t^T \mu_s ds$$

and finally

$$\bar{\psi}_t = \arg \min_{\psi > 0} H(t, \bar{\phi}_t, x, \psi; \bar{p}_t) = -\frac{1}{\gamma}(\bar{p}_t \wedge 0) = \frac{1}{\gamma} \left( u'(F_T - \bar{\phi}_T) - \int_t^T \mu_s ds \right)^+$$

where the terminal value  $\bar{\phi}_T$  is the solution of the equation

$$\phi = \frac{1}{\gamma} \int_0^T dt \left( u'(F_T - \phi) - \int_t^T \mu_s ds \right)^+. \quad (9)$$

It is easy to see that under our assumptions, this equation admits a unique solution which is strictly positive.  $\square$

*Example 2.* To obtain an explicit solution, assume that  $\mu_s \equiv \mu < 0$  is constant, and that the penalty function is quadratic:  $u(x) = \frac{\kappa}{2}x^2$ . A straightforward computation then gives the solution to equation (9).

$$\bar{\phi}_T = \begin{cases} F_T - \frac{\frac{\mu T^2}{2} + \gamma F_T}{\kappa T + \gamma}, & \frac{\mu T^2}{2} + \gamma F_T \geq 0 \\ F_T - \frac{\mu}{\kappa^2}(\gamma + \kappa T) - \sqrt{\frac{\mu^2}{\kappa^4}(\gamma + \kappa T)^2 - \frac{2\mu}{\kappa^2} \left( \frac{\mu T^2}{2} + \gamma F_T \right)}, & \frac{\mu T^2}{2} + \gamma F_T < 0 \end{cases} \quad (10)$$

In the first case,  $\bar{\phi}_T \leq F_T$  and the optimal trading strategy satisfies

$$\bar{\psi}_t = \frac{-\frac{1}{2}\kappa\mu T^2 + \gamma\kappa F_T - \mu\gamma T}{\gamma\kappa T + \gamma^2} + \frac{\mu}{\gamma}t \geq 0, \quad \forall t \in [0, T].$$

In other words, the agent trades continuously between time  $t = 0$  and  $t = T$ , at a linearly decreasing rate. The expected gain of the power producer is

$$G_{exact}(T) = \mathbb{E}[P_T]F_T - \left[ \frac{\gamma\lambda}{2}T + \mu\lambda T^2 + \frac{\mu^2}{3\gamma}T^3 + \frac{\kappa}{2} \left( \frac{\frac{\mu T^2}{2} + \gamma F_T}{\kappa T + \gamma} \right)^2 \right]$$

where  $\lambda = \frac{-\frac{1}{2}\kappa\mu T^2 + \gamma\kappa F_T - \mu\gamma T}{\gamma\kappa T + \gamma^2}$ .

In the second case, we have  $\bar{\phi}_T > F_T$ . Introduce the time

$$t^* = T - \left( T + \frac{\gamma}{\kappa} \right) + \sqrt{\left( T + \frac{\gamma}{\kappa} \right)^2 - \frac{2}{\mu} \left( \frac{\mu T^2}{2} + \gamma F_T \right)}. \quad (11)$$

The optimal strategy is given by

$$\bar{\psi}_t = \frac{\mu\gamma + \sqrt{\mu^2\gamma^2 - 2\mu\kappa\gamma(\kappa F_T - \mu T)}}{\gamma\kappa} + \frac{\mu}{\gamma}t,$$

for  $t \in [0, t^*]$  and  $\bar{\psi}_t = 0$  for  $t > t^*$ . In other words, the agent trades continuously at a linearly decreasing rate until time  $t^* < T$  and then stops. The expected gain of the power producer is

$$G_{exact}(T) = \mathbb{E}[P_T] \cdot F_T - \left[ \frac{\gamma\lambda}{2} t^* + \mu\lambda(t^*)^2 + \frac{\mu^2}{3\gamma}(t^*)^3 + \mu(T - t^*)\bar{\phi}_T + \frac{\kappa}{2}(F_T - \bar{\phi}_T)^2 \right]$$

$$\text{where } \lambda = \frac{\mu\gamma + \sqrt{\mu^2\gamma^2 - 2\mu\kappa\gamma(\kappa F_T - \mu T)}}{\gamma\kappa}$$

*Remark 3.* The solution in the case when no forecast is available can be obtained by replacing the penalty function  $u$  with the average penalty

$$\tilde{u}(x) = \mathbb{E}[u(F_T - \mathbb{E}[F_T] + x)]$$

in Proposition 5. The strategy is completely deterministic in this case since contrary to the situation without market impact, there is no lump-sum trade at the terminal date.

**Continuous forecast updates** In the presence of market impact, since the trading strategy is necessarily continuous-time, we assume that the forecast is updated in continuous time as well. To solve this problem, introduce the value function

$$w(t, \phi, x) = \min_{\psi \geq 0} \mathbb{E} \left[ \int_t^T \phi_s \mu_s ds + \frac{\gamma}{2} \int_t^T \psi_s^2 ds + u(F_T - \phi_T) \middle| \phi_t = \phi, X_t = x \right].$$

The following proposition can be obtained using standard tools of stochastic control (see Propositions 4.3.1 and 4.3.2 in [9]). These standard results do not guarantee the uniqueness of the viscosity solution, because the set of controls is not bounded, but if the strategy  $\psi$  is restricted to a bounded domain, uniqueness follows easily from Theorem 4.4.5 in [9].

**Proposition 6.** *The value function  $w(t, \phi, x)$  is a viscosity solution of the Hamilton-Jacobi-Bellman equation*

$$\min_{\psi \geq 0} \left\{ \frac{\gamma}{2} \psi^2 + \frac{\partial w}{\partial \phi} \psi \right\} + \phi \mu_t + \frac{\partial w}{\partial t} + \frac{1}{2} \sigma_t^2 x^2 \frac{\partial^2 w}{\partial x^2} = 0$$

or equivalently,

$$-\frac{1}{2\gamma} \left( \frac{\partial w}{\partial \phi} \wedge 0 \right)^2 + \phi \mu_t + \frac{\partial w}{\partial t} + \frac{1}{2} \sigma_t^2 x^2 \frac{\partial^2 w}{\partial x^2} = 0$$

for  $\phi \geq 0$ ,  $x \geq 0$  and  $t \in [0, T]$  with the terminal condition

$$w(T, \phi, x) = u(f_{prod}(x) - \phi).$$

**The case when both buy and sell transactions are allowed** In the final paragraph we consider the situation when the agent can both buy and sell electricity. In this case, for the quadratic penalty function, the optimal strategy is explicit (since  $\mathcal{A}^{ac}$  is a linear space) and described by the following proposition. For the non-quadratic penalty function the optimal strategy may be found by solving the HJB equation as above.

**Proposition 7.** *Assume that the penalty function is quadratic:  $u(x) = \frac{x^2}{2}$ . Then the optimal trading rate satisfies*

$$\psi_t^* = \frac{\mathbb{E}[F_T|\mathcal{G}_t] - \phi_t^* - \frac{1}{\gamma} \int_t^T ds (\gamma + T - s) \mu_s ds}{\gamma + T - t}.$$

*Proof.* The first order condition writes

$$\mathbb{E} \left[ \int_0^T dt \xi_t \left( \int_t^T \mu_s ds + \gamma \psi_t - (F_T - \phi_T) \right) \right] = 0$$

for every  $\mathbb{G}$ -adapted process  $\xi$ . Therefore,

$$\gamma \psi_t + \int_t^T \mu_s ds = \mathbb{E} [F_T - \phi_T | \mathcal{G}_t], \quad (12)$$

which means that the left-hand side is a martingale. This in turn means that for  $s \geq t$ ,

$$\mathbb{E}[\psi_s | \mathcal{G}_t] = \psi_t + \frac{1}{\gamma} \int_t^s \mu_u du.$$

Substituting this formula into (12), we then obtain

$$(\gamma + T - t) \psi_t = \mathbb{E}[F_T | \mathcal{G}_t] - \phi_t - \frac{1}{\gamma} \int_t^T ds (\gamma + T - s) \mu_s ds.$$

□

### 4.3 Numerical illustrations

In this section, we illustrate the optimal trading policies for a large producer, determined in section 4.2, with numerical examples.

In these examples, we assume that the trading takes place continuously over  $T = 6$  days, that  $\mu = -0.2$  (this means that the forward price per MWh decreases by 1 euro every 5 days as one approaches maturity),  $\gamma = 4800$  (liquidating 0.1MWh over 1 hour has a cost of approximately 1 euro), the daily volatility of  $(X_t)$  is  $\sigma_t \approx 27\%$  (that is, 66% over the 6 days; this corresponds roughly to the estimated value for one of the power plants we studied in this paper), and that the penalty function is  $u(x) = Px^2$  with  $P = 100$  (this means that with, e.g., 0.1MWh volume mismatch, the extra price to pay is 1 euro).

To obtain the numerical examples, we first solve the HJB equation by finite differences, and then simulate random trajectories of the forecast process with a fixed value

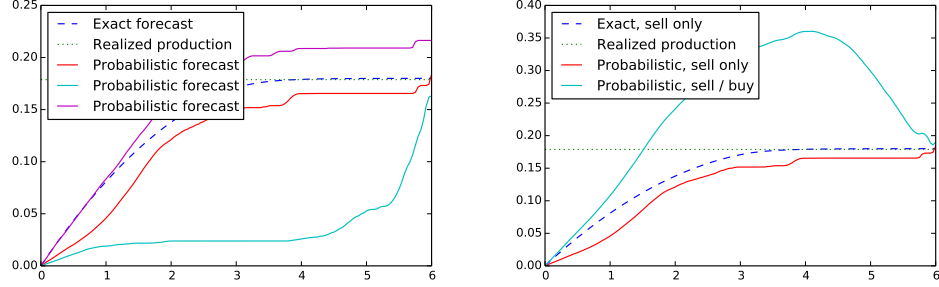


Figure 10: Sample selling strategies with market impact. Strategies are updated dynamically as new information becomes available. The left graph shows three strategies where sales only are allowed. The right graph shows one strategy with sales only and one with both buy and sell transactions.

of realized production. For these trajectories, we compute the corresponding trajectories of the optimal trading strategy  $\phi_t$ . Figure 10 shows several sample trajectories when only selling is allowed (on the left graph) and when both buy and sell transactions are permitted (right graph).

Finally, we compute the realized penalty (the value of the expression under the expectation in (8)) corresponding to the simulated trajectories of the forecast process and the optimal trading strategy, with the objective of evaluating the economic value of the optimal strategy in different contexts. Figure 11, left graph compares the distribution of the realized penalty with volatility  $\sigma = 66\%$  and that with volatility  $33\%$ . One can see that with the lower volatility, the premium for early trading compensates the cost of market impact and the volume penalty, leading to negative overall penalty for most of the trajectories, whereas for the higher volatility, the penalty is positive for most trajectories. Figure 11, right graph, quantifies the impact of allowing both buy and sell transactions (the volatility was taken to be  $66\%$  for both experiments). One can see that once again, if the agent is allowed to both buy and sell, the premium for early trading compensates the volume penalty and the cost of market impact.

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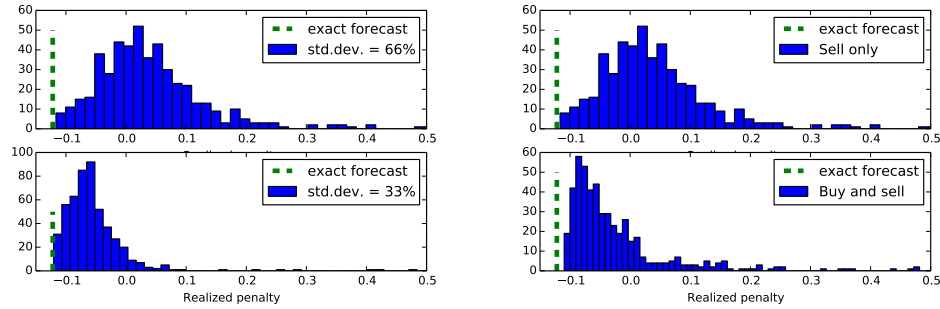


Figure 11: Left: Realized penalty for different forecast quality. Right: realized penalty with and without buy transactions.

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